The Few-Body Physics group applies and develops state-of-the-art methods for attacking quantum mechanical problems with two, three, or more bodies within atomic, molecular and subatomic physics. The projects are sufficiently diverse to allow students at all levels to enter the research front either at the Bachelor’s or Master’s level. We also offer short-term (5 ECTS) research project on an ad hoc basis.

Some of the recent projects have been studies of low-dimensional atomic systems, quantum magnetism in strongly-interacting 1D systems, polarons in 1D, three-body decay processes in ultracold gases (related to experiments in the Jan Arlt group), three-body nuclear physics of heavy-heavy-light systems, etc. Below we present some examples of our recent results in both nuclear and atomic physics using few-body methods.

The Three-Body Problem

The objective is to solve the Schrödinger equation, \( H \psi = E \psi \), for systems that have three constituents. This is far more complicated that for two-body problems, yet it is also far more interesting! One such effect is that of ‘Borromean binding’ in three dimensions. This can be explained by considering three identical bosonic particles. Then one may ask whether two of the three can form a bound state, i.e. have a negative two-body energy, \( E_{BB} \). Surprisingly, it turns out that in the limit where \( E_{BB} = 0 \), not only do you find three-body bound states, but you find infinitely many state with \( E_{BB} < 0 \).

Few-body problems with many particles

A recent focus has been the emergence of cluster structures in nuclear physics, i.e. the fact that particular subsystems in a nucleus may be more bound than others. An example is illustrated in Fig. 1. Here we see a heavy subsystems with many nucleons interacting with two single nucleons. An effective Hamiltonian will now have the form

\[
H_{YY} = -\frac{\hbar^2}{2m} \nabla^2 + V_{c} + V_{s} + V_{c} + V_{L} + V_{LBD}.
\]

Here we have interaction terms between the large core (denoted by \( c \)), and the two single nucleons (denoted by 1 and 2). We also have three-body force, \( V_{s} \), which acts whenever all three constituents are close to each other. What we have achieved is an effective model for a many-body problem that can now be treated with few-body methods.

**Figure 1:** A many-body system that can be considered as one heavy particle and two light particles.

When solving the Schrödinger equation \( H \psi = E \psi \), we obtain wave functions which are functions of \( r_{c} \) and \( y \), in Fig. 1, and also the center-of-mass but it is decoupled and can be ignored here. In Fig. 2 we show a three-body wave function, \( \psi_{c,pp} \), for the case of a large core interacting with two protons. These solutions have applications in nuclear decay of light- and medium-mass nuclei. In particular, they can be very important in astrophysical settings and knowledge of the three-body quantum states is decisive for models of how the elements are formed in stars.

**Figure 2:** Probability distribution for a system consisting of a core, \( c \), and two protons, \( p \). Here \( r_{cp} \) is the radial distance between the two protons, and \( r_{cp} \) is the distance from the center-of-mass of the two protons to the core (the norm of the vector \( r \)).

Magnetic effects in one dimension

One-dimensional quantum systems have some peculiar properties. Amazingly, there are actual model that can be solved exactly in 1D and this is used as a benchmark to test other analytical and numerical methods. Recently, we have developed new methods to attack strongly interacting bosonic systems in 1D when these systems are confined by an external trap. In this case all of the old exact methods no longer apply and new tools are demanded.

**Figure 3:** a) Energy spectrum of a four-body system with two \( A \) bosons and two \( B \) bosons in a harmonic trap as function of the short-range repulsion between \( A \) and \( B \) parameterized by a strength, \( g \). b) Configuration space for the coordinates of a four-body wave function. The coordinates are at the bottom.

In Fig. 3 we show an example of a four-body system (two \( A \) and two \( B \) particles, both of bosonic nature). When removing the center-of-mass one can write all the configurations of the system in a 3D phase-space such as that shown in Fig. 3b). The energies of such a system can be computed with numerical methods, but we have also managed to produce a (semi)-analytical result for the limit of large interaction (where \( 1/g \sim 0 \) in Fig. 3a). It is of interest to note that for strongly repulsive particles the ground state is an example of a purely ferromagnetic quantum system, i.e. the \( A \) and \( B \) particles are always found in either an \( AABB \) or a \( BBAA \) ordering along the spatial dimension of motion. It is very rare to have exact results for a purely quantum ferromagnetic effect.

**Figure 4:** a) A sketch of the doubly-degenerate ground state for the 1+8 system. Panels b)-d) show the \( AA \) pair-correlation function.

An important problem in solid-state and condensed-matter physics is that of an impurity moving around in a bath of particles with which it interacts. In 1D this is particularly important as many widely applied approximate theoretical tools (for instance mean-field theory) do not work. The single impurity interacting with a system of Bose particles is called the Bose polaron problem. We have recently developed a new method for solving this Bose polaron in 1D when the system is trapped as it is the case in many experiments. It turns out that one again finds a very clear separation of impurity and Bose system which indicates a strong ferromagnetic tendency in the system.

**Supervisors**

Aksel S. Jensen (ajs@phys.au.dk)
Dmitri Fedorov (fedorov@phys.au.dk)
Nikolaj T. Zinner (zinner@phys.au.dk)

**Ph.D. Students**

Dennis Hove (dennis@phys.au.dk), three-body problems in nuclear physics
Amin S. Delchaoueh (amin@phys.au.dk), trapped 1D system and quantum magnetism