Content:

We develop hyperbolic geometry in the Poincaré half-plane. The group of isometrics is $P - SL(2, \mathbb{R})$ acting as transformations $z \to \frac{az+b}{cz+d}$. Riemann surfaces are obtained as quotients of h by discrete subgroups Γ of $P - SL(2, \mathbb{R})$, such as the modular group $P - SL(2, \mathbb{Z})$. The Γ-automorphic Laplacian is a self-adjoint operator. For a compact Riemann surface the spectrum is discrete, in the non-compact case such as for the modular group there is also a continuous spectrum. The spectral theory of Laplacians on Riemann surfaces combines algebra, geometry and analysis and is important in analytic number theory.