Two famous problems in number theory

3-4 hours of lectures per week.

Lecturer
Alexei Venkov

Content
In the first part of the course I will explain the introductory material in order to show the proof of Goldbach's ternary problem that every sufficiently large odd integer can be represented as the sum of three odd primes $n = p_1+p_2+p_3$. It was mentioned by Goldbach in a letter to Euler, dated 7 June 1742, that every number is the sum of two odd primes. This Goldbach's binary problem is still open (unproved). In the course I will follow the proof of Linnik, based on previous results of Hardy and Littlewood, relating ternary problem to extended Riemann conjecture and density theorems for zeros of zeta-function and L-series.

In the second part of the course, I will consider the Gauss problem of behavior of the class number function $h(D)$, where D is a discriminant of a quadratic field $Q_D = Q(\sqrt{d})$, $h(D) \to \infty$, $D \to -\infty$. We know that in Q_D the decomposition of an integer in a product of primes is not unique. If $h(D) = 1$, then the prime factorization of integers in the classic sense holds. Gauss also conjectured that there are only 9 imaginary quadratic fields with $h = 1$, $d = -1, -2, -3, -7, -11, -19, -43, -67, -163$. These two parts of Gauss conjecture were proved in the 20th century. In the course I will follow the proofs of Hecke, Heilbronn and Stark. The behavior of $h(D)$ for $D \to +\infty$ is unkown. Also, it is unkown: are there infinitely many Q_D, $D > 0$, with $h(D) = 1$? This is also part of Gauss' conjecture on the class number function.

Prerequisites
This is a continuation of my course on Riemann zeta-function. But new students are also welcome if they are familiar with courses on complex function theory and *Algebra 1*.

Literature
H. Davenport, *Multiplicative number theory*, Chicago, 1966.
Z. I. Borevich and I. R. Shafarevich, *Number theory*, Academic Press, 1964, (ch. 5).
Wang Yuan (Ed.), *Goldbach conjecture*, World Scientific, 1983.