Morse theory

4 hours a week, 2 hours lectures + 1-2 hours seminar.
We expect the participants to give seminar talks!

Lecturer
Marcel Bökstedt and *Johan Dupont*.

Content
The idea of Morse theory is to analyse a manifold through the level sets of a given smooth function on it (assuming suitable condition on the critical points of the function).

We will eventually discuss a recent version of this idea that uses it to construct a cell decomposition of the manifold. This will involve simplicial sets, and the important construction of the classifying space of a category.

This approach has led to spectacular results on manifolds, especially but not exclusively in the 50's and 60's. These results are still important, but not the main focus of our course.

An extension of the idea that has been very profitably used from the 50's up to the present is in the study of *infinite dimensional* spaces with properties similar to manifolds.

One special case of this is the action integral $E(\gamma) = \int_0^1 \mid \gamma'(t) \mid^2 dt$ defined for closed differentiable curves on a Riemannian manifold. A "critical point" of the function E on the infinite dimensional "manifold" of closed paths is actually a geodesic curve on the Riemannian manifold. Morse theory works here, and it gives a decomposition of the space of closed curves on the manifold which is related to the closed geodesics.

The scheme has been vastly generalized. The space of solutions of some differential equations can be considered as the space of critical points of an integral. If you are lucky and/or clever, this leads to a decomposition of the space of all functions, related to the space of solutions to the equation. For instance, the Yang-Mills functional is related in this way to a moduli space of stable holomorphic bundles.

For a short introduction to the field (and to a few of the important mathematicians who have worked on it), see :

Raoul Bott, *Morse Theory Indomitable*, in Publication mathématique d'IHES 68, (1988).

There is also an introduction to Morse theory in chapter 12 of Ib Madsen, Jørgen Tornehave, *From Calculus to Cohomology.*

Prerequisites

Differentiable manifolds to the level of *Topology 1.*

One or more of the courses *Geometry 2* or *Homology and Homotopy* will be very helpful. Some elements from both will be used, but they are formally not prerequisites.

Literature

In addition to articles and lecture notes, we will patch from the following two classics:

John Milnor, *Morse Theory*, Ann. of math. studies number 51, Princeton University press, 1963.

John Milnor, *Lectures on the h-cobordism Theorem*, Princeton university press, 1965. This book seems to be out of print, but it is available as a scanned pdf file at: http://www.maths.gla.ac.uk/ ajb/btop/bt-resources.html