Elliptic functions

3-4 hours of lectures per week.

Lecturer
Alexei Venkov

Content
The theory of elliptic functions is an important part of mathematics. It was created in the 17 Century, and it was developed during the time by many outstanding mathematicians, including Euler, Legendre, Gauss, Abel, Jacobi, Eisenstein, Kronecker, Weierstrass....
Historically, the word elliptic came from the theory of elliptic integrals, which occurs in many problems, for example, in the computation of the length of an arc of an ellipse. The integral is of the form

$$\int \mathbb{R}(x, y) dx$$

where R(x,y) is a rational function in x and y, and y^2 is a polynominal of 3 or 4 degree of x without multiple roots.

Elliptic functions can be defined as inverse functions of elliptic integrals. In the course we will study general properties of elliptic funtions and will consider important examples of Weierstrass $P(z), \sigma(z), \zeta(z)$ functions and Jacobi theta functions. Since elliptic functions parametrize elliptic curves, we will study also modular functions including the Eisenstein series, Dedekind ζ-function and Klein absolute J-invariant. As the applications of the theory we will prove some classical problems in number theory, the law of quadratic reciprocity for Gaussian sums (Jacobi symbols), the theorem on the representation of a number as a sum of four squares. (More generally, the representation of a number by a quadratic form). We will derive also the Kronecker limit formula for quadratic field and we will see aproaches to famous 12 Hilbert problem (Kronecker's Jugendtraum) which still remains open nowadays.

Prerequisites
The knowledge of complex functions theory and elementary number theory.

Literature

1) K. Chandrasekharan, Elliptic functions, Springer 1985.

2) A. Weil, Elliptic functions according to Eisenstein and Kronecker, Springer 1976.

3) Yu. I. Manin, Von Zahlen und Figuren, Preprint 2002.

4) My notes (A. Venkov)

Evaluation
Students who do not intend to take a degree in Mathematics or Statistics from the University of Aarhus, but wish to earn credits for a 2.dels course from the Department of Mathematics, should indicate at the beginning of the course that they wish to be examined.

The form of examination for these students will be active participation together with oral or written contributions.

Credits
10 ECTS

Semester
Spring 2003