Introduction to the Riemann Zeta function

3 hours of lectures per week.

Lecturer
Erik Balslev

Content
Riemann in his 8 page paper on the prime number theorem from 1859, introduced his Zeta function as an analytic extension to the whole complex plane of the function defined for $Res > 1$ by

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \frac{1}{1-p^{-s}}, p \text{ prime}$$

with a simple pole at 1.
The development of an explicit formula for the distribution of primes led him to study the zeros ρ of $\zeta(s)$ with $0 \leq \operatorname{Re}\rho \leq 1_-$. He indicated the precise asymptotics for $T \to \infty$,

$$\#\{\rho \mid 0 \leq \operatorname{Im}\rho \leq T\} = \frac{T}{2\pi}\log\frac{T}{2\pi} - \frac{T}{2\pi} + O(\log T)$$

This as well as the prime number theorem were proved about 40 years later.

In this course we go through Riemann's paper as well as later modifications and proofs of the prime number theorem, the factorization of $\zeta(s)$ over the zeros and the asymptotic distribution of zeros.

Prerequisites
Complex Function Theory

Literature
Edwards: Riemann's zeta function

Evaluation:
The course is evaluated passed/failed.

Credits:
10 ECTS

Semester:
Spring 2003